Limitations in Thermal Scale Modeling

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Nomenclature

c_p = specific heat at constant pressure, Btu/lbm-°F
d = characteristic thickness, ft
h = joint conductance, Btu/hr-ft²-°F
k = thermal conductivity, Btu/hr-ft-°F
L = characteristic length, ft
q = energy dissipation per unit volume, Btu/hr-ft³
q' = energy dissipation, Btu/hr
R = scale model ratio
S = energy flux per unit area, Btu/hr-ft²
t = time, hr
T = absolute temperature, °R
$[T]$ = parameter temperature dependance
U = parameter uncertainty (tolerance)
V = parameter value
α = coefficient of thermal expansion
α_s, ϵ = solar absorptivity and emissivity, respectively
ρ = density, lbm/ft³
ω = solid angle, steradians

Subscripts

m,p = scale model and prototype, respectively
pe = probable error

Superscript

$*$ = ratio of model/prototype properties (i.e., $k^* = k_m/k_p$)

Introduction

The basic similitude criteria for the radiation-conduction system of an unmanned spacecraft can be developed either from dimensional analysis or from the differential equations necessary to describe the behavior of the system under consideration. The latter technique is preferred, when such equations are available, since it can give more direct insight into the physical behavior of the system. Extensive studies of scale modeling criteria and their application to spacecraft have been presented.[1–6] Limitations on the maximum size, and hence the maximum scale ratio of thermal scale models, will most often result from the size of the space chamber and the solar simulator that is to be utilized. Minimum limits on scale model ratio can result from available materials, material gages, and material properties. However, the most significant limit on the smallest feasible scaling ratio is due to the desired prediction accuracy.

Experimental errors are present in all thermal tests, regardless of scale, and will not influence the limiting scale ratio. However, the accuracy of scaled model test data is determined by the errors or uncertainties associated with the design and fabrication of the model and the accuracy of the environmental simulation. Factors involved are due to materials, fabrication costs, fabrication practices, and test instrumentation. The desired material composition and gage may be unavailable, or thermophysical property values may be uncertain. Fabrication errors introduce variances in dimensions and solid angles. Model temperature errors can be caused by thermocouple location accuracy, instrumentation heat leaks, and instrumentation lead gometry.

By considering the limitations of scale modeling,[7] we can better define the boundaries of the application of thermal modeling and provide the thermal designer with some basis for a judgment about its application to a particular problem. Identification of inherent difficulties and limitations in the techniques will also serve to focus future research on the more difficult areas.

Errors in Thermal Scale Modeling

The general scale modeling criteria[1–6] are summarized in Table 1 in cartesian coordinates for both the general three-dimensional and two-dimensional geometric distortion cases. This Note is concerned with the temperature preservation technique (as opposed to material presentation technique), and the simplified equations that form its basis are also presented in Table 1.

If the solution to the differential equations and their boundary conditions were available (either in closed form or as a numerical network solution) it would be of the form

$$T = f(k,\rho c_p,h,\alpha_s,\epsilon,L,d,q',S) \qquad (1)$$

The difference in temperature resulting from violations of the scaling criteria could be calculated from

$$\Delta T = f(x_i) - \bar{f}(x_i) \qquad (2)$$

where x_i are the individual parameters of Eq. (1), $\bar{f}(x_i)$ is evaluated for the parameters of the model as it was built (with compromises and violations of the scaling criteria), and $f(x_i)$ is evaluated for the parameters of the model as it should have been built (in accordance with the scaling criteria).

A closed-form solution is generally not available. A numerical solution could be perturbed to obtain the correction factors as required to adjust the model experimental data. Alternatively, the range of correction factors could be estimated from the formulations of the general scaling criteria presented in Table 1. This linearized technique utilizing appropriate formulations of the generalized scaling criteria may be used to estimate the range of errors due to violations and compromises of the scaling criteria, but a perturbed numerical analysis is required to evaluate the magnitude of the actual error at this node.

There are errors inherent in scale modeling as a result of uncertainties in 1) material properties, 2) geometric dimensions, and 3) test conditions. The sources of many of these errors are listed in Table 2.

Scale models are usually built such that joint resistance is negligible. The remaining uncertainties in steady-state scale modeling studies have been identified and probably errors

Table 1 Thermal scale modeling criteria[a]

General criteria		Temperature preservation criteria	
Cartesian coordinates 3-dimensional	Cartesian coordinates 2-dimensional	Cartesian coordinates 3-dimensional	Cartesian coordinates 2-dimensional
$k^* = L^*T^{*3}$	$k^* = L^{*2}T^{*3}/d^*$	$k^* = L^* = R$	$k^* = L^{*2}/d^*$
$t^* = L^*(\rho c_p)^*/T^{*3}$	$t^* = d^*(\rho c_p)^*/T^{*3}$	$t^* = L^*(\rho c_p)^*$	$t^* = d^*(\rho c_p)^*$
$q^* = k^*T^*/L^{*2}$	$q^* = k^*T^*/L^{*2}$	$q^* = 1/L^* = 1/R$	$q^* = 1/d^*$
$S^* = k^*T^*/L^*$	$S^* = d^*k^*T^*/L^{*2}$	$S^* = 1$	$S^* = 1$
$h^* = k^*/L^*$	$h^* = k^*/L^*$	$h^* = 1$	$h^* = L^*/d^*$

[a] Note: 1) assumes radiative properties preserved, 2) assumes proportional temperature dependence for thermal conductivity and capacity.

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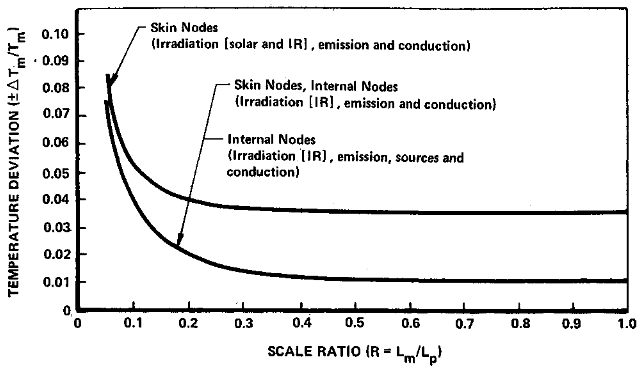

Fig. 1 Probable temperature error in thermal scale modeling as a result of uncertainties.

estimated in Table 3. The probable errors estimated are based upon the ranges of tabulated thermophysical properties values available in reference handbooks and properties measurement and testing experience with Boeing facilities.

A statistical error analysis based upon probable error[8-10] results in the equation

$$\Delta T_{pe} = \left[\sum_{i=1}^{n} \left(\frac{\partial T}{\partial x_i}\right)^2 (\Delta x_i)^2{}_{pe}\right]^{1/2} \tag{3}$$

The $(\partial T/\partial x_i)$ terms have been calculated using the general scaling criteria presented in Table 1. The probable errors calculated for several typical model nodes are presented in Fig. 1.

An examination of the figures indicates that the nonuniformity of solar illumination within the chamber is the major source of probable error for scale ratios above 0.2. Below a $\frac{1}{10}$ scale ratio the effects of uncertainties in thermal conductivity and the effect of a constant geometric tolerance dominate the calculation of probable error. The rapid increase of probable error below the $\frac{1}{10}$scale ratio indicates that only the most carefully conducted studies will obtain useful results for these small-scale ratios. Accurate thermal conductivity measurements and tightened shop tolerances could result in accuracies greater than indicated by this study, but only at greatly increased cost.

Table 2 Sources of error

Materials	Test environment
Thermal properties	Energy dissipation
k: V, $[T]$, U	Power leads
h: V, U	Space simulation
c_p: V, $[T]$, U	Background temperature
ρ: V, U	Solar beam
α: V, U	Intensity
Radiative properties	Collimation
α_s: V, (T), U	Spectral match
ϵ: V, $[T]$, U	Instrumentation
Dimensions	Location
L: V, U	Tolerance
d: V, U	Response time
ω: V, U	Thermocouple leads

Table 3 Estimate of errors inherent in scale modeling

Parameter	2σ Range of error	Probable error
Radiative properties, α_s,ϵ	±0.02	±0.0067
Solar flux, Btu/hr-ft²	±4.42	±1.99
Thermal conductivity, Btu/hr-ft°R	±1.0	±0.337
Geometric tolerances, ft	±0.0026	±0.00088
Internal dissipation, Btu/hr	$\pm 0.01Q_m$	$\pm 0.00337Q_m$
Thermocouples, °R	±3.0	±1.01

Conclusions

Limitations are inherent in thermal scale modeling techniques as a result of uncertainties in material thermal properties, model dimensions, instrumentation effects, and environment simulation. These limitations indicate a realistic lower limit on scale modeling of $\frac{1}{5}$ to $\frac{1}{7}$ scale. An absolute lower limit on the order of $\frac{1}{10}$ scale seems appropriate. The lower the scale ratio the more difficult, time consuming, and expensive will be the resulting model. These limitations appear reasonable, based on the assumptions of this study, but the reader is reminded that the lower limit on scaling is strongly a function of original model size and the manufacturing tolerances for which the modeler is willing to pay.

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